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Linguistic and cultural factors in the readability of mathematics texts: the Whorfian hypothesis revisited with evidence from the South African context

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ABSTRACT

South Africa is a country of many languages and cultures. Education is mostly in English which implies that about 80% of all secondary school students are second language learners. Currently many mathematical problems are posed in real-life contexts. This not only introduces more language in mathematics texts, but also more issues related to culture. When considering the influence on readability, it is difficult to disentangle the role of linguistic and cultural factors and this in turn seems to be a manifestation of the Sapir-Whorf hypothesis. The study was therefore guided by research questions such as: what linguistic and cultural factors in the ordinary language of mathematics texts influence readability? Protocol analysis was used with students aged 17–18 years to establish readability problems. Linguistic as well as cultural factors were identified and the hypothesis that improved readability of mathematics assignments improves achievement was confirmed in a number of cases. Writers of mathematics texts should therefore consider linguistic as well as cultural factors when writing for different reading audiences. Other conclusions address teaching and examination practice.

RÉSUMÉ

Facteurs linguistiques et culturels dans la lisibilité de textes mathématiques: reprise de l'hypothèse de Whorf avec des données provenant du contexte de l'Afrique du sud

L'Afrique du sud est un pays aux langues et cultures multiples. L'enseignement est effectué principalement en anglais, ce qui implique que 80% des élèves de toutes les écoles secondaires sont des apprenants en langue seconde. Beaucoup de problèmes mathématiques sont posés couramment dans des contextes de vie réelle. Cette pratique fait pénétrer plus de langue dans des textes mathématiques mais aussi plus de questions liées à la culture. Quand on considère ce qui influe sur la lisibilité, il est difficile de séparer le rôle des facteurs linguistiques et culturels, et ceci semble bien être alors relever de l'hypothèse de Sapir-Whorf. Notre recherche a par conséquent été guidée par des questions telles que: quels facteurs linguistiques et culturels ont un effet sur la lisibilité dans le langage ordinaire des textes mathématiques? On a effectué une analyse de protocoles d'étudiants âgés de 17–18 ans pour voir ces questions de lisibilité. On a identifié des facteurs linguistiques aussi bien que culturels et l'hypothèse qu'une meilleure lisibilité des tâches mathématiques permet de meilleurs résultats a été confirmée dans nombre de cas. Les rédacteurs des textes mathématiques devraient par conséquent prendre en considération des facteurs linguistiques aussi bien que culturels quand ils écrivent pour des publics différents. D'autres conclusions se rapportent à l'enseignement et aux pratiques d'évaluation.

INTRODUCTION

Traditionally, readability has been measured by linguistic surface elements, such as word and sentence length (Flesch, 1948; Markel, 1992). Recently, more sophisticated, computerised tools have been developed that focus amongst other factors on lexical density and sentence complexity (see article by Harrison and Bakker in this issue). In the past decade however, several pleas have been made to take into consideration linguistic factors in readability assessments – factors that focus on the interaction between reader, writer and text (see Ulijn and Strother, 1991 and 1995, for an overview of available experimental studies on the matter as well as a discussion of their results). These psycholinguistic factors need to be considered, in particular in the case of texts which are intended for a specific readership, as texts for study and professional purposes often are. If readers have to process such a text in a language other than their native tongue the linguistic aspects become even more complex, quite apart from other cultural issues.

Mathematics secondary school examinations in South Africa are a subject of discussion. If examinations are in English, they have to be written and read by pupils whose native tongue is English (E1), Afrikaans (E2), or native African languages (E3). The cultures involved are Western/European (E1, E2 and of the mathematics discipline itself as an expression of western thought) and African (E3). In the readability of such texts it is difficult to disentangle the role of linguistic and cultural factors. This seems to be an illustration of the Sapir-Whorf hypothesis (Sapir, 1949 and Whorf, 1956) which holds, even in its weak version, that culture frames language and language frames culture: they are intertwined. However, recent developments in mathematics teaching make it possible to single out at least one part of that language-culture connection. Whereas ordinary language in traditional mathematics was mainly restricted to some logical connectors structuring the argument based upon formulae, the ordinary language context of many recent mathematics assignments brings everyday life – as well as culture – directly into focus.

The experimental study reported here made it possible to assess 3 situations of a language-culture connection in the text of the mathematics assignments of South

African school examinations: 1. Non-verbal version (N), hardly any ordinary language, only the culture factor of mathematics itself; 2. Original version (O) with natural language (and culture context). If those texts are processed by the above groups (E1, E2 and E3), the possible differences in scores between E1 and E2 may be attributed (all other things being equal) to exclusively the extra difficulty of the second language in the case of E2, since both groups share the same (Western/European) background culture. The differences with E3 however, can to a large extent be attributed to diverse linguistic as well as cultural factors. 3. In an effort to reduce the effect of linguistic factors in the readability of those assignments, a protocol study was launched with bright students (in mathematics) of the E1, E2 and E3 groups to get formulations with an increased readability: the adapted version (A). Since the 'cultural' content of the assignments was not changed, differences in score might be more the result of the remaining cultural factors.

The South African context provided an excellent opportunity to revisit the ideas of Whorf and Sapir. South Africa is a country of many languages and cultures. At school level about 80% of all pupils are African. Secondary education is either in English or Afrikaans which means that most secondary school students are second language learners (Afrikaans is a language closely related to Dutch). This language practice has various implications for mathematics education considering the important role language plays in the developing of mathematical concepts. Moreover, many mathematical problems are currently posed in real-life contexts. This not only introduces more ordinary language in mathematics texts, but also more issues related to culture. The past 15 years' reading research has suggested that less skilled readers use context and that the hallmark of skilled reading is fast context-free word identification and rich context-dependent text understanding (Perfetti, 1995). One might imagine how much a black student of mathematics might be handicapped: (s)he is forced to learn mathematics by taking in more unfamiliar contexts than before.

This study tries to manipulate linguistic and cultural factors (as dependent variables) in the readability of mathematics texts from the perspective of the reader (E1, E2 and E3) and of the text (NOA) with the score (1–10) of 9 assignments as the independent/experimental variable. This way we tried to answer the following overall research question: *To what extent is the readability of mathematics texts affected by linguistic and cultural factors and as an applied corollary of this: how could the readability of South African secondary school examination assignments be increased for all groups of pupils?* This is in a way a verification of the Whorfian hypothesis in its weak version: language and culture are related also in the professional field of mathematics.

READABILITY OF MATHEMATICS TEXTS: LINGUISTIC AND CULTURAL FACTORS

What does the concept of readability in mathematics text imply and how can we differentiate the more linguistic factors from the more cultural ones? The majority of researchers have identified readability with comprehensibility (Selzer, 1983). For the sake of this study readability is defined as *the ability of the text to communicate the intention of the writer to the intended reader*. According to this approach factors other than those reflected in the superficial formulation of text contribute to readability. To understand the factors that influence readability is no easy task. Readability formulae based upon surface linguistic elements, such as number of syllables in a word or number of words in a sentence do not seem to be able to guide writers in the production of more readable mathematics texts. A closer look at literature suggests that readability be approached from both a psychological (reader/writer) and a deeper linguistic point of view.

Linguistic factors are mainly related to different levels of the text: vocabulary, syntax and discourse. However, readability is also influenced by the *interaction* between reader, writer and text. Therefore, to understand what makes one text more readable than the next, one should also focus on the reading process and investigate the mind of the reader which gives the concept of readability a rather psycholinguistic dimension.

When reading mathematics texts, the reader will use his/her background knowledge to comprehend the semantic content. Background knowledge is the linguistic equivalent for what cognitive theorists call schemata or mental constructs. According to schema theory the human mind constructs abstract schemata on the basis of experience (Huckin, 1983). The richer the experience the richer the schemata. The power of schemata resides in the ability they give the reader to induce inferences from written information. Readers with rich schemata do not need so much explicit detail because a single word can activate a large amount of unwritten information in the working memory (or short term memory) of the brain. Successful communication between reader and writer is based on shared schemata – on those available to the non-specialist partner of the communication process. For example, if a student has had no academic experience of vectors, mathematics text referring to velocity and displacement might cause many problems due to unshared schemata between reader and writer.

Although most studies on second language communication focus on the lexical component of text, other issues like syntax and organization of discourse play an important role in the logical development of an argument. Structural issues should therefore have an important influence on mathematical reasoning. Furthermore, mathematics text normally contains a certain amount of ordinary language together with portions of the mathematics register. (Linguistically speaking, a register can be considered to be a subset of the total language.) Textual issues influencing readability can therefore be related to either ordinary language or elements of the mathematics register. Both first and second language readers of mathematics texts have to cope with difficulties in the mathematics register. However, the difficulties for second language readers are increased by the fact that the mathematics register is used simultaneously with an ordinary language they have not yet fully mastered. Moreover, the writer in the communicative situation of mathematics school examinations is a hidden, but very powerful person with a more extensive background knowledge, a superior competence regarding the mathematics register and first language proficiency in English. This all may produce readability problems for the examinee. Comprehension problems could be aggravated even more if differences between reader and writer were extended to the cultural level. This is almost always the case in a country like South Africa. Most writers of mathematics texts are either English- or Afrikaans-speaking which implies a dominant Western culture while most students at school are African.

Reading in a second language is not only a matter of language acquisition. It is also a matter of learning another culture. There seems to be no language without a culture and no culture without a language. However, Witherspoon (1980), in a review of 20 studies, reports that whereas culture is in language and language is in culture, culture seems to affect oral communication more than written language. To many African students, not only home and school *languages* differ, but also home and school *cultures*. Schools are inclined to be organized according to Western patterns of thinking and doing – patterns which are often different to African behavioural patterns. For instance, Africans' orientation towards time is simply not the same as Europeans' (Usunier, 1993). Traditionally Africans are not time-driven and as a result one seldom sees an African in a hurry. From a cultural point of view, a traditional African student could experience difficulties in a time-restricted and/or time-driven examination. Time-pressure in examination conditions almost always causes anxiety which in turn affects readability: there is a need for more information and yet one tends to see less. To discuss the influence of culture on the comprehension of mathematics texts, two manifestations of culture will be considered: the relationship between *culture and language* and *culture and schemata*.

Culture, language and schemata

In this study 'culture' relates to what Hofstede calls patterns of thinking, feeling and potential acting – patterns which are learnt throughout a person's lifetime (Hofstede, 1991:4). The Sapir-Whorf hypothesis links language to human thinking and therefore in a sense to culture. It suggests that the structure of a language strongly influences or even fully determines the way its native speakers perceive the world. This hypothesis has endured much criticism, but even its weaker form predicts that language and culture influence each other mutually. According to the weaker form, languages would not differ not so much in what can be said in them, but rather as to what is relatively easy to say (Au, 1983:156). For example, the English language expresses hypothetical situations by means of hypothetical constructions more easily than Chinese. A study of African languages reveals that hypothetical constructions do not exist in most of these languages whereas this type of construction is often used in mathematics texts. Consider the following verbal problem:

> There are exactly enough exercise books in a class to supply each pupil with the same amount of books. If there had been five more students each would have received two books less and there would have been six books left over. If however there had been three students less each would have received one book more and there would have been eleven books left over. How many students are in the class and how many books are available?

The mathematical difficulty of the above question is increased by the hypothetical construction of sentences. African students should find the comprehension of this question more difficult than most first language readers of English.

Interesting research linking culture and language, is reported by Kaplan (1980) and Clyne (1981). They experienced that second language students who were brought to the level of proficiency necessary for writing text, wrote texts which differed in important ways from the texts written by native speakers of English. Kaplan refers to these different rhetorical structures as *cultural thought patterns*. In the meeting of European and African cultures in South Africa, English/Afrikaans writers would probably follow linear constructions: the sequence of ideas is direct, very much in line with Hall's concept of explicitness (Hall and Hall, 1989). African discourse is more likely to tend towards circularity considering the topic from different angles – a more indirect approach with freedom to digress and introduce 'extraneous' information. Therefore, text produced by English/Afrikaans writers could be less accessible to African readers due to the Western 'cultural pattern' of the discourse. Another contribution to this issue (Ulijn and Salager) reports evidence of the cultural factor in academic discourse in general. Ulijn (1996) and Ulijn and Campbell (1997) demonstrate cultural effects on structuring/paragraphing and reading instructions in brochures intended for industrial coffeemakers. These effects might apply to mathematics discourse as well.

Another point to remember is that the culture of a people is not only reflected in the way they use language. The *messages* communicated by that language often have cultural roots as well. The schemata of cultural experiences are more deeply rooted than the schemata of any other experience. They have an historical background and have constantly been shaped since childhood. More specifically we could consider culture as part of the cognitive system or long-term memory of the reader and cultural schemata as short-cuts for readers to obtain a quick understanding of 'culture-bound' texts. The most extensively studied schemata are those called 'story schemata'. Story schemata are frequently dependent on cultural influences. When assessing the realistic approach to mathematics education one notices that a verbal problem can often be seen as a short 'story' with a specific cultural background. Cultural schemata have conventional story structures known to the reader of that culture and therefore it is important that not only the context, but also the *behaviour* within the context be familiar to the reader. Only if the behaviour is compatible with the reader's cultural experiences can comprehension of the context be optimized. For African culture, Lasisi et al (1988) demonstrates the effect of a cultural schema: 300 Yoruba speakers (in Nigeria) understood a culturally related reading text significantly better than a foreign-based one, both texts being in English. The same research (on fairytales) however showed a superior understanding of foreign-based texts in Yoruba than in English. It seems that culture and language are related, but they may also operate separately.

Referring to mathematics text, one could say that mathematical formulae also communicate a certain cultural behaviour: the behaviour of variables relative to one another. If writers of mathematics text describe a verbal problem within a real-life context and were to use a mathematical formula which generates behaviour that is not in line with a reader's cultural schemata, comprehension difficulties are bound to occur.

Overall the research seems to suggest that cultural factors have an important influence on the readability of mathematics texts. Is this true?

THE RESEARCH PROBLEM

During the last few years educators have become increasingly aware of the important role reading and language play in the successful accomplishment of mathematical tasks – especially amongst second language learners (Mulder, 1991; Terlouw, 1991; Lagerwerf, 1992). More and more ordinary language is presently being used in mathematics texts and therefore also in assessment tasks. The reason for the increase in language is mainly due to the shift from the ordinary manipulation of numbers to the more important area of problem solving. Problems are posed within real-life contexts, but real-life problems have to be communicated verbally. Whereas a non-verbal mathematical problem can be posed in an international, precise language – a language students are expected to learn – a verbal problem has to be set in a language that takes the linguistic and cultural aspects of the reading audience into consideration. When writing for large audiences this is a difficult commission. Text with a high degree of accessibility for one reader may not be so easy to understand for the next. For example, an everyday context like a factory may not be part of the everyday life of students from remote rural areas and therefore a mathematics text describing a problem within the context of a factory may cause comprehension difficulties to certain readers. There is a surmise that readability factors in the ordinary language of mathematics texts often cause unnecessary comprehension difficulties to students which could in turn even influence achievement (Adetula, 1990; Weerman, 1994). If this were true for first language readers, the impediment experienced by second language readers could be even greater.

The issue of second language readers is especially relevant to a country like South Africa. As indicated, the majority of secondary school students are black and receive their secondary education in a language which is not their mother tongue. Furthermore, African languages are in no way related to the two languages of instruction, Afrikaans and English. Most black students prefer English. For black students, formal education in English starts from the fifth school year, but reports on the increase of their language proficiency in English are not very promising. Most African students do not live in an English environment and most of their fellow students at school are also Africans which means that the natural medium of communication between these students is one of the African languages. To many, therefore, conditions to acquire a high level of proficiency in English are not very encouraging. In South Africa it is noticeable how few black students study mathematics. Compared to 73% white or first language students, only about 33% black students take mathematics in their final school year and the success rate of this group is well below 40% (Strauss et al. 1993). Naturally there could be various reasons for this phenomenon, but many educators are convinced that the language issue is one of the main problems in black education (Swanepoel, 1989; Macdonald, 1990).

Why two experimental studies?

After all is said and done, it is not quite clear how students themselves experience the readability of mathematics texts. Reports tend to be of empirical studies done on reading comprehension of expository prose in ordinary English. It is not known how these factors influence the readability of mathematical English. It could be that mathematical formulae and other parts of the mathematics register carry enough information to overcome comprehension difficulties caused by the ordinary language of the text. Moreover, where research results did involve mathematics, it was mainly limited to primary and junior secondary students. It could be that older students have developed a good enough mathematics proficiency to compensate for bothersome readability factors, as has been demonstrated by Ulijn and others for electro-technical and computer science texts (Ulijn and Strother, 1995). It is also not known if readability problems could have a significant influence on achievement levels. The best way to find answers to these and other related issues seemed to be to go to the students themselves. Initial research questions were reformulated and extended to include the following:

- What readability factors in the ordinary English of mathematics texts cause comprehension difficulties to senior secondary students?
- Will improved readability of test questions, improve students' achievement levels?
- What cultural factors influence readability?
- Who will gain most by improved readability – first or second language readers?

Answers to these questions are important. Achievement in mathematics to a great extent motivates students to opt for mathematics related careers. Any factor that unnecessarily hinders achievement in mathematics is not only bad from an educational point of view, but is also bad for the economic and technological growth of a country. It would therefore be important for a writer to know what makes one text more accessible than the other. Two studies were therefore initiated. First a protocol study was launched to find answers to the research questions. Analysis of students' protocols not only revealed various readability problems, but also generated the main hypothesis which was tested in the experimental study. It is possible to give only a representative sample of both method used and results obtained, but this should be sufficient to evidence the main argument of the article (for full details, please refer to Prins, 1995).

PROTOCOL STUDY

The protocol study was inspired by the work of Newell and Simon (1972) who used think-aloud protocols to analyse human problem-solving strategies. A think-aloud protocol is a type of verbal report in which students speak out their thoughts and behaviour without any interference from an outside observer. Here one should mention the methodological limitation of such an approach because some criticism could be raised against the use of verbal reports as reliable data. They are most informative when students are aware of difficulties and how they go about solving them. However, automatic processes are not easily verbalized and therefore not readily available for study. Fortunately, it is exactly this limitation that makes think-alouds so suitable for identifying readability problems.

Because students are more conscious of comprehension difficulties they are more able to talk about them. Apart from being able to talk about the specific comprehension problem, students usually revert to comprehension *strategies* as soon as they experience readability problems. Block (1986) confirms that most of these strategies are easily verbalized and include strategies like re-reading, re-formulation, a slower reading rate or translation into the mother tongue. Ericsson and Simon (1980) contend that verbal reports, elicited and interpreted with care, are a valuable and reliable source of information.

Subjects, materials and method

In this study, subjects were students aged 17–18 years in their final school year. Three different language groups were involved: a first language group (the E1 group) and two second language groups. One second language group had Afrikaans as first language and will be referred to as the E2 group while the other second language group comprised African students and will be referred to as the E3 group. The big difference between the two second language groups was the relation of their mother tongue to English. Afrikaans is a language with Germanic (Dutch) roots and therefore related to English, whereas the African students' mother tongues are inherent to Africa and in no way related to the English language. E3 is not to be understood as English as a 3rd language and E2 not always as English as a second language: it is possible that subjects might have had more than one mother tongue other than English. However, English is the most important vernacular to communicate among most ethnic groups, at least between South Africans of European and African descent.

There were six students in each language group and all of them were high achievers in mathematics. It was necessary to use students with a high proficiency in mathematics to help the researcher distinguish between a mathematical and a readability problem when listening to the think-alouds. Students answered nine assignments (also referred to as nine problems) and all of them were examples of so-called verbal problems. (Verbal problems are known as such due to the fairly large amount of verbal language used to describe the mathematics problem.) The specific nine verbal problems were chosen because of potential readability difficulties identified by the researcher. They represent typical examination questions used for assessment purposes in pre-tertiary mathematics examinations in South Africa. Assignments were set in English for all language groups.

Students did the think-aloud protocols individually. They were asked to read and think aloud as they solved the nine problems. All think-alouds were recorded on tape. After students had completed the think-aloud experiment, they were asked to adapt the nine assignments to a more comprehensible form. They could change anything as long as the mathematical difficulty of the questions remained the same. Students did the adaptations within 24 hours of the think-aloud experiment and handed the adapted assignments back in written form. Adaptations came mainly from E1 and E2 readers. E3 readers remarked they could not adapt some of the questions because they did not understand them. (See Appendix for original and adapted versions of Assignment 3.)

Readability problems were identified by carefully listening to the protocols on tape. Any disturbance in the normal reading rhythm was taken as an indication that there was an interference in the ease of comprehension. Disturbances were signalled by a variety of interruptions, like re-reading, a stumbling over words, heavy breathing, a change in tone of voice or even remarks of discontent. Sometimes it was necessary to look at the student's adaptation to make sure whether the student had actually experienced the problem as a *readability* problem. Most of the readability guidelines reported in the literature were confirmed by either students' think-alouds or their adaptations. One of the research questions asked how readability problems would affect achievement. It was therefore important to have an indication of the students' achievement levels for the nine assignments. Students were therefore asked not only to think aloud while solving the verbal problems, but also to write down their solutions. All solutions were marked according to a previously set memorandum.

Results and discussion of think-aloud protocols

For the sake of this paper the report will focus mainly on results generated by the analysis of African readers' protocols. Although the think-alouds revealed that all three groups encountered readability problems, African readers (the E3 group) experienced most problems more intensely. Increased anxiety was clearly audible on the tapes of E3 readers and readability problems at times even caused complete communication breakdowns – something that never happened to the other two groups. As the analysis of protocols progressed, it became clear that E3 readers found the mathematics text less accessible than their E1 and E2 counterparts not only because of linguistic reasons, but also because of cultural issues. To discuss these aspects in more detail the readability problems have been divided into the following five categories: Difficult vocabulary; Text structure; Obscure information; Visualization difficulties; Non-verbal factors.

The discussion will be restricted to only a few examples coming mainly from the original versions of Assignments 3 and 5 (see Appendix). These examples should illustrate the nature of each category. To a large extent Assignments 3 and 5 are good representations of all nine assignments because in Assignment 3, knowledge of differential calculus is needed whereas Assignment 5 is a question based on linear programming. Linear programming and differential calculus were the mathematical themes in most of the other assignments as well.

Difficult vocabulary

This refers to unfamiliar or difficult words or phrases. Although difficult vocabulary is problematic for all kinds of readers, in mathematics it causes more problems for second language readers. The protocols confirmed that second language readers are more often unable to discern whether the meaning of a difficult word is absolutely necessary for solving a mathematical problem. Listening to the think-alouds also confirmed that most E3 readers process information bottom-up and a difficult word often hinders a global conceptual analysis or recognition of relationships between variables.

Assignment 5 had a few words and phrases that caused comprehension problems even for first language readers; namely, *utilized*, *profit margins*, *optimal search line*, *percentage daily capacity*. This specific assignment has six sub-sections and section 5.3 caused much anxiety to E3 readers. For example, after spending 47 minutes on this assignment, one E3 reader could not get further than section 5.3. (Only 16 minutes are allocated for a question like Assignment 5 in authentic examination conditions.) The rather difficult language of the otherwise relatively easy section 5.3, caused a complete comprehension breakdown. She kept on saying: *'I don't understand the question... Oh, I don't know... Oh, I'm taking too much time'*. After another 13 minutes she despondently said: *'I'll come back to this one later if I have enough time.'* Needless to say she never came back.

Many students could not answer section 5.6 correctly either – mainly because of comprehension problems caused by the phrase *'percentage daily capacity'*. One student reread the question a few times and then said: *'Mayo, Mayo! I really don't know what's going on here ... I'll do my best... I don't understand the question'*. After a while he left the question undone.

The think-alouds not only confirmed the need of the E3 group for more plain language, but also the need for more time to read and process information. Need for more time could be because of the relative weaker language proficiency in English, but the need could also have a cultural basis. Some of the E3 think-alouds took more than twice as long to complete than those of the other second language group – the E2 group.

Text structure

This category refers to problems related to the overall organization of text, whether in sentences or overall discourse. The think-alouds verified the importance of structural issues, especially for second language readers (Kieras, 1978). In Assignment 3, the composition of the text has in a sense violated the principle of hierarchical progression by inverting the order of importance. The irrelevant, redundant information concerning 8000 calculators is given in the prominent first sentence, whereas the crucial information that the selling price refers to only *one* calculator is reserved for the inferior last position of the discourse. When listening to the think-alouds it was remarkable to hear how only a few readers read this last bit of very important information. More E3 readers tripped over this hurdle than E1 or E2 students.

Another important structural issue causing comprehension difficulties was the issue of cultural thought patterns. When comparing the analysis of the think-alouds with the students' adaptations it became clear that often the linear thought patterns of predominantly English or Afrikaans writers, made the text less accessible for E3 readers. Compare the following original and adapted version of Assignment 9. The specific adapted version was suggested by an E3 student.

Assignment 9: Original version

Two straight roads intersect perpendicularly at O. P is a point on one road such that OP = 10 km. Two persons, at P and O respectively, start to walk simultaneously, the one at P in the direction of O along the one road at three km/h, and the other at O along the other road away from O at 4 km/h. After t hours they reach the positions A and B on the roads.

9.1 Find an expression for AB^2.
9.2 After how many hours are the persons nearest to one another?
9.3 What is the shortest distance between them?

Assignment 9: Adapted version

2 Straight roads intersect at right angles at a point O. P is a point on one of the roads so that the distance of OP = 10 km. 2 people – one at P and the other at O, start to walk at the same time, the person at P walking towards O on one road while the person at O walks on a different road away from point O. The person at P walks with a speed of 3 km/h while the person at O walks with a speed of 4 km/h. After a time of t hours they reach positions A and B on these two different roads.

9.1 Find an expression for the distance AB.
9.2 After what length of time are the 2 persons closest?
9.3 What is the shortest distance of AB?

When comparing the adapted version with the original assignment, one notices that the adapted formulation is inclined to have a circular structure resembling the cultural thought patterns of the E3 reader's mother tongue. Not only is the information more descriptive and more linguistically explicit, but it is also more repetitive and has a recurring nature.

Obscure information

Information of this kind is not easily understood and causes uncertainty within the reader. There were different reasons for obscurity. One such reason was cultural interference. This happens when the described context is not part of the student's cultural experiences. In Assignment 3, the concept of *profit* was not well known to African students from rural areas because in these parts of the country much trading is done by exchanging of goods. Part of the empirical research was done in a rural area and personal conversations with students confirmed that to many, profit was not a common concept.

Cultural experiences form part of readers' mental schemata that help them gain access to text more easily. When these cultural schemata are missing, no activation of contexts can take place. One could of course argue that mathematics is part of a wider education and should expose students to experiences that reach further than their immediate surroundings. This is true, but if these contexts are used in examination conditions for the first time, the text should not be so condensed as for instance in Assignment 3. Visuals could be used to aid comprehension or other ways of formulation could elucidate less known contexts.

Visualization difficulties

Information that is either too abstract or too condensed makes it difficult to form an image of the communicated context. Normally readers find sentences that are easy to visualize easier to understand than low-imagery sentences (Holyoak, 1974). This is especially true for second language readers who are more dependent on concrete information. In Question 5 the rather abstract and condensed information, does not allow students to form an idea of the type of product or the kinds of departments to which the examiner is referring. In one talk-aloud an E3 student remarked, *'Too many figures and numbers; it gets very confusing.'*

When reading English mathematics texts, it seems that Africans would need more descriptive and more concrete information – not only because of uncertainty due to a second language, but also because of the cultural characteristic of their mother tongue. Traditionally, Africans use language very descriptively, giving much detail. For example, in one African language a numerical value like 2,367 can freely be translated as: thousands which are two, hundreds which are three, tens which are six and units which are seven. Mathematics text on the other hand is usually very bare, giving very little unnecessary information.

Non-verbal factors

This category refers to letter symbols or mathematical formulae used in such a way that it interferes with the processing of information. One of the sub-divisions of this category was *entangled verbal/non-verbal information*. An example of a mathematics assignment communicating this kind of information is Assignment 3. One of the sentences reads:

> The cost of n calculators is C = 100n − 200 and n calculators can be sold at a price of P = 400 − 0.02n per calculator.

Apart from the obscurity caused by the missing money unit, this type of language interfered with reading rhythm as well as comprehension ease and was specifically mentioned by students during the personal interviews. Most of them experienced this type of language as unnatural or unreal. Referring to questions like Assignment 3, one of the students said: *'If you want to give us real-life problems, why don't you give them to us in real-life language?'*

When re-reading the original questions one realized that many of the verbal problems were already semi-mathematized. The examiner was communicating a real-life situation in a type of short-hand language which was quite natural to him, but difficult for students to process. Although one would think that this semi-mathematized form would help students to move more easily to the fully mathematized form, this 'aid' seems to interfere with comprehension.

The issue of entangled verbal/non-verbal information can prove difficult for writers of mathematics text. Students who are specialist readers of mathematics could prefer data presented in semi-mathematized form. At school however most students are not specialized readers of mathematics yet – they do not speak mathematics 'like a native' (Pimm, 1989:1). Especially second language readers have a double hurdle – neither the ordinary English of the text nor the non-verbal mathematical symbols are part of their everyday language. An unnatural mixture of the two makes comprehension even more difficult.

Frequency of readability problems

The few above examples are intended to illustrate the nature of each of the five categories. Needless to say, students experienced many more readability problems. Table 1 (overleaf) provides a summary of the readability problems identified during the analysis of students' protocols. The table covers all nine assignments. A certain readability problem was counted only once – even if more students in a specific language group vocalised the same comprehension difficulty.

The fact that much less data was available on the tapes of E3 readers, makes it rather difficult to compare the results of these readers with the other two groups. There is a suspicion that because of their weaker fluency in English, E3 readers did not vocalise so much of their thinking as the other readers. One does however notice that the rank orders of all three groups are more or less the same except for the more linguistic category, *difficult vocabulary*. Furthermore the table seems to suggest that structural problems are not much of a problem for second language readers because the table shows the rank order for both E2 and E3 groups is fairly

Table 1. Number of readability problems vocalised per category per language group.

Category	E1		E2		E3	
	T	R.O.	T	R.O.	T	R.O.
Difficult vocabulary	5	5	20	2	15	3
Structural problems	13	3	7	4	11	4/5
Obscure information	33	1	30	1	32	1
Visualization difficulties	11	4	6	5	11	4/5
Non-verbal factors	16	2	19	3	16	2

T: Total number of readability problems verbalized per category.
R.O.: Rank order (1 = vocalised most frequently; 5 = vocalised least).

low in this category. Listening to the think-alouds one realises this is not true. Although structural problems were not vocalised as often as for instance difficult vocabulary, structural problems had a serious influence on comprehension: in at least two of the nine questions they terminated the problem solving processes of E3 students. Students found it difficult to recognise structural problems.

Formulating the hypothesis for the experimental study

As mentioned before, a test score was allocated to all students' solutions. On average, students' scores were 20% lower than their average school performance. Although readability factors could have been responsible for the lower scores, one was not sure whether the level of mathematical difficulty was not the main reason for the students' poor performance. At school, assessment tasks do not contain *only* so-called verbal problems, as was the case in the text used for the protocol study. Usually students find verbal problems exceptionally difficult, especially if problems demand students to translate the verbal data into mathematical formulae – a process known as mathematization. With this in mind, the following hypothesis was formulated:

> Improved readability of the ordinary language in mathematics examination questions will improve achievement.

Adaptations of students as well as information generated by the protocols were used to improve the readability of the original nine assignments and design a composite test which was used to test this hypothesis.

EXPERIMENTAL STUDY

The protocol study was rather exploratory in nature. The purpose of the following experiment was to verify whether the adaptations, as suggested by the students who did the protocol study, would improve the readability of the questions to such a degree that the improvement would be reflected in other students' test scores.

Method

More than 300 students, representing E1, E2 and E3 readers, served as subjects for this experiment and wrote a composite test with varying degrees of readability. Students represented 12 schools across the country, from rural as well as suburban areas. Beforehand the students were subjected to standardised Mathematics and English proficiency tests. Selection of students, whose test scores were later used for the statistical analysis, was carried out according to the following criteria: (a) a scholastic achievement level between 55% and 75%, and (b) a score between 6 and 9 (out of 10) for the Mathematics and English proficiency tests.

The rationale behind the first criterion came from the protocol study. Analysis of protocols predicted that students above the 75% level would not gain so much by improved readability. A high subject proficiency usually helped students to overcome many of the comprehension difficulties experienced in the talk-aloud experiment. It was expected that test scores of students below the 55% level would not gain significantly by improved readability either. Their mathematics was not strong enough so although improved readability might have made the text more accessible, the mathematics required to solve the problem was still too difficult for them. The criteria for the two standardised tests were to help with the matching of students from the different schools. After the above criteria had been applied to all those who had written the composite test, results of 108 students could be used, 36 per language group.

The composite test contained the same nine assignments used in the protocol experiment, but assignments were posed in three different versions: original, adapted and non-verbal. The original versions were the same versions as those used in the protocol experiment. The adapted version was the same verbal problem as the original version, but with improved readability. The non-verbal version had no reference to any context so virtually no verbal language was used as illustrated in the non-verbal version of Assignment 5 (see Appendix). The reason for including the non-verbal version was to ascertain to what degree students could execute the necessary algebraic computations. Most marks in examinations are allocated in this area.

Students were attributed at random to one of three different sets of tests. Each test-set consisted of 3 original (O), 3 adapted (A) and 3 non-verbal versions (N) across the 9 assignments. Each student therefore answered an equal number of each version for each assignment to avoid an order effect. Set 1 started with OAN, that is an original version of Assignment 1, an adapted version of Assignment 2, a non-verbal version of Assignment 3 and so on. Set 2 started with ANO and set 3 started with NOA. The statistical significance of the differences between the test scores for the original and the adapted versions of each question was assessed by using the non-parametric Mann-Whitney U-test. Significance was tested for the combined language group as well as for each language group separately. The hypothesis that improved readability of questions will improve achievement was confirmed in a number of important cases.

Results

For the E3 group, improved readability improved test scores of Assignments 3 and 5 by 27% and 13% respectively. One must remember that the nine assignments differed – not only regarding readability, but also in the mathematics. Therefore one cannot expect improved readability to have the same effect on test scores of all assignments. The results of Assignment 3 are shown in Figure 1.

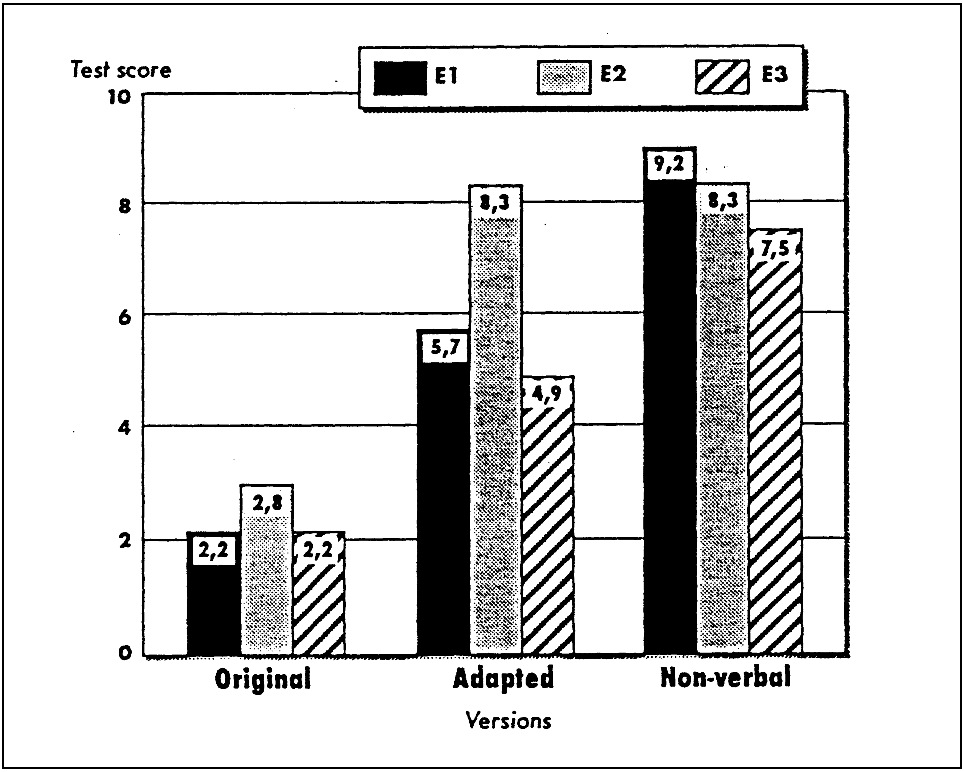

Figure 1. Achievement level per version (Assignment 3).

All groups performed extremely poorly on the original version of this question and excellently on the non-verbal one. Major problems of the original version of Question 3 were redundant information, entangled verbal/non verbal information and inappropriate mathematical formulae. Except for inappropriate formulae, all these were removed in the adapted version. E3 students were also handicapped by the culturally biased context referring to profit, but this could not be removed without changing the nature of the mathematical problem. (For the purpose of the experiment it was necessary to keep the mathematical difficulty of the problem the same.) This was also the reason why the same mathematical formulae were retained in the adapted version.

Differences between the original and adapted versions of Assignment 3 were significant for all groups so the hypothesis was confirmed in all cases. To form a more general idea of the effect of improved readability on achievement, one could consider the total improvement of test scores per language group across all nine questions. Table 2 shows the results. Differences in test scores were once again analysed using the Mann-Whitney U-test.

Table 2. Average percentage score of all nine questions for the original and adapted versions.

Group	Average % for nine original versions	Average % for nine adapted versions	% Gain	p <
E1 (n = 36)	49	61	12	0.02
E2 (n = 36)	51	70	19	0.001
E3 (n = 36)	41	55	14	0.002
E1 + E2 + E3 (n = 108)	47	62	15	0.001

If one were to accept the above results of the original and adapted versions coming from two examination papers, an 'original' and an 'adapted' one, then one could conclude that students who had written the adapted paper performed significantly better than the other group. On average, improved readability improved test scores of African students by 14%. Overall the two second language groups (E2 and E3) gained more by improved readability than the E1 group. A total graph, including the average test score for the nine non-verbal versions, is shown in Figure 2.

One of the important results emerging from the composite test was the equal performance of all language groups on the non-verbal versions. This has special significance for the E3 group. Whereas these students had the weakest performance for the original and adapted versions, on average they performed equally well on the non-verbal versions. This indicates that these students had the same computational skills as their E1 and E2 counterparts, but that factors related to language and culture inhibited their performance.

CONCLUSIONS

What can be concluded from the results of the protocol and experimental studies for the Whorfian hypothesis as related to the influence of language and culture on the readability of mathematics texts? Furthermore, what would the results imply for mathematics teaching in general and more specifically to mathematics teaching in South African schools?

Linguistics and cultural effects on readability

To sum up, even with the attempt of the protocol study to minimise the effect of linguistic and cultural factors on the readability of mathematics assignments, these effects, to a certain extent, still 'survived' in the experimental study. E1 readers had the least linguistic and cultural problems while E2 readers had to cope with more linguistic difficulties than their E1 counterparts. E3 readers on the other hand experienced the most linguistic barriers as well as an extra burden caused by culture related problems. The non-verbal version enabled readers with an African cultural

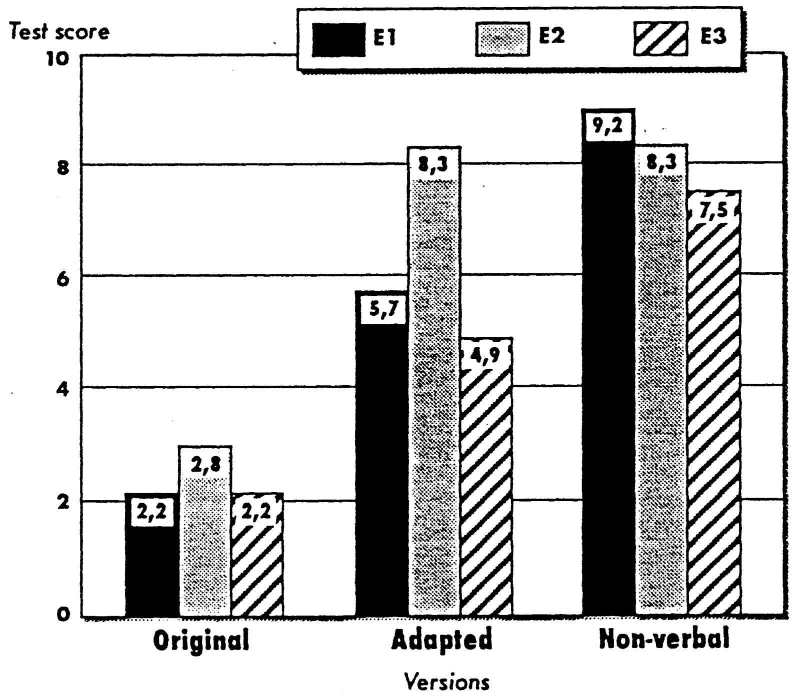

Figure 2. Achievement level per version averaged over all 9 assignments.

background to overcome at least the Western cultural effects of the ordinary language situation. Although the adaptations following the protocol study did help, E3 readers were still hampered by the cultural baggage of the ordinary language for both the original and adapted versions. Their knowledge of the mathematics register could not always compensate for problems caused by ordinary language and the culture behind this. Therefore results of this study can be considered as a support for the Whorfian hypothesis in its weak form – culture frames language and language frames culture – in the field of mathematics.

It remains difficult to disentangle linguistic and cultural effects on readability, especially if one views much of mathematics as an expression of Western culture. On the other hand, culture can be considered as learned behaviour. Therefore, if mathematics is seen as universal thought, bright students of mathematics can and will always emerge from any cultural background, provided they receive appropriate mathematics teaching based upon sound readability of texts.

Implications for mathematics education

The implications of this study have resulted in various suggestions for teaching and examination practice (Prins, 1995). Referring to the South African situation, and more specifically to E3 learners, an obvious suggestion is that all students should receive mother tongue education up to at least pre-tertiary level. However, in South Africa the political and economic climate makes it difficult to implement this idea (NEPI, 1992). Another suggestion would be to provide separate examination papers for distinctly different cultural groups at pre-tertiary level (all students in South Africa write a national examination at pre-tertiary level). Examination papers should have the same mathematical difficulty, but differ as regards context and language. If this suggestion is not practically possible, it seems feasible that pre-tertiary examination papers should be bilingual for all cultural groups. Examination papers for E3 readers could be in the language of instruction as well as the language of the student's mother tongue. Referring to E3 readers, the specific mother tongue is usually one of the official African languages of the region. Furthermore, it would be imperative that examination papers be composed by a group of examiners who share the same culture and language as examinees. This would ensure that subtle linguistic and cultural factors be used to the advantage of the readers.

The study has identified some of the numerous factors that should be taken into account when writing for optimum readability of mathematics texts. Although writers cannot be held responsible for the language and mathematics proficiency of students, they do have the responsibility to consider their reading audience with care and write accordingly.

REFERENCES

ADETULA, L. (1990) Language factor: does it affect children's performance on word problems? *Educational Studies in Mathematics*, **21**, 351–365.

AU, TERRY KIT-FONG (1983) Chinese and English counterfactuals: the Sapir-Whorf Hypothesis revisited. *Cognition*, **15**, 155–187.

BLOCK, E. (1986) The comprehension strategies of second language readers. *TESOL Quarterly*, **20**, 463–494.

CLYNE, M. (1981) Culture and discourse structure. *Journal of Pragmatics*, **5**, 61–66.

ERICSSON, K.A. and SIMON, H.A. (1980) Verbal reports as data. *Psychological Review*, **87**, 215–251.

FLESCH, R. (1948) A new readability yardstick. *Journal of Applied Psychology*, **32**, 221–233.

FREUDENTHAL, H. (1983) Major problems of mathematics education. In M. Zweng, et al. (Eds.) *Proceedings of ICME 4*. Boston: Birkhäuser.

HALL, E.T. and HALL, M.R. (1989) *Understanding Cultural Differences: Germans, French and Americans*. Yarmouth, ME: Intercultural Press, Inc.

HOFSTEDE, G. (1991) *Cultures and Organizations: software of the mind*. England: McGraw-Hill.

HOLYOAK, K.J. (1974) The role of imagery in the evaluation of sentences: imagery or semantic factors? *Journal of Verbal Learning and Verbal Behaviour*, **13**, 163–166.

HUCKIN, T.N. (1983) A cognitive approach to readability. In P.V. Anderson, C.R. Miller and J. Brockman (Eds.) *New Essays in Technical and Scientific Communication*. New York: Baywood Press, pp. 90–108.

KAPLAN, R.B. (1980) Cultural thought patterns in inter-cultural education. In K. Croft (Ed.) *Readings on English as a Second Language*. Massachusetts: Winthrop Publishers, pp. 399–418.

KIERAS, D. (1978) Good and bad structure in simple paragraphs: effects on apparent theme, reading time and recall. *Journal of Verbal Learning and Verbal Behaviour*, **17**, 13–28.

LAGERWERF, B. (1992) Taalproblemen. *Nieuwe Wiskrant*, **10**, 36–41.

LASISI, M.J., FALODUN, S. and ONYEHALU, A.S. (1988) The comprehension of first and second language prose. *Journal of Research in Reading*, **11**, 26–35.

MACDONALD, C.A. (1990) *Crossing the Threshold into Standard Three in Black Education*. Consolidated Main Report of the Threshold Project. Pretoria: HSRC Publishers.

MARKEL, M.H. (1992) *Technical Writing: situations and strategies*. New York: St Martin's Press.

MULDER, F. (1991) Een uur heen, twee uur terug. *Nieuwe Wiskrant*, **11**, 70–75.

NATIONAL EDUCATION POLICY INVESTIGATION (NEPI). (1992) *Language*. Cape Town: Oxford University Press.

NEWELL, A. and SIMON, H.A. (1972) *Human Problem Solving*. Englewood Cliffs, NJ: Prentice Hall.

PERFETTI, CH.A. (1995) Cognitive research can inform reading education. *Journal of Research in Reading*, **18**, 106–115.

PIMM, D. (1987) *Speaking Mathematically: communication in mathematics classrooms*. London: Routledge.

PRINS, E.D. (1995) *The Influence of Readability on Achievement in Senior Secondary School Mathematics: a study on verbal problems with special reference to second language readers*. Unpublished Doctoral Thesis, University of Stellenbosch, South Africa.

SAPIR, E. (1949) *Culture, Language and Personality*. Berkeley: University of California Press.

SELZER, J. (1983) What constitutes a readable technical style? In P.V. Andersen et al (Eds.) *Essays in Technical and Scientific Communication: research, theory, practice*. New York: Baywood Publishing Co. Inc, pp. 71–89.

STRAUSS, J.P. et al (1993) *Education and Manpower Development*, vol. 13. Bloemfontein: Universiteits-drukkery.

SWANEPOEL, P.H. (1989) *Special Language Issues in Black Education*. Pretoria: Unisa.

TERLOUW, P. (1991) *Wiskunde in Intercultureel Perspektief*. Leeuwarden: Educatief.

ULIJN, J.M. (1996) Translating the culture of technical documents: some experimental evidence. In D. Andrews (Ed.) *International Dimensions of Technical Communication*. Washington: Society of Technical Communication Press, pp. 60–86.

ULIJN, J.M. and CAMPBELL, C.H. (1997) International paragraphing: what can cultures learn from each other in the communicative and managerial field? In M. Pütz, B. Smieja and M. Tasch (Eds.) *Human Contact through Language and Linguistics*. Frankfurt: Peter Lang Verlag, pp. 491–500.

ULIJN, J.M. and STROTHER, J. (1991) The ties that bind writing research to readability: who writes the most readable texts? In C. Harrison and E. Ashworth (Eds.) *Celebrating Literacy: defending literacy*. Oxford: Blackwell, pp. 75–92.

ULIJN, J.M. and STROTHER, J.B. (1995) *Communicating in Technology and Business: from psycholinguistic theory to international practice*. Frankfurt: Peter Lang Verlag.

USUNIER, J. (1993) *International Marketing: a cultural approach*. London: Prentice Hall International Ltd.

WEERMAN, F. (1994) Over talen – en wiskunde knobbels. *Euclides*, March 1994, 167–172.

WHORF, B.L. (1956) *Language, Thought and Reality*. London: Chapman and Hall.

WITHERSPOON, G. (1980) Language in culture and culture in language. *International Journal of American Linguistics*, **46**, 1–13.

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APPENDIX

Original version (O): Assignment 3	Adapted version (A): Assignment 3
A manufacturer has the capacity to produce 8000 calculators of a certain type per week. The cost of producing n calculators is $C = 100n + 200$, and n calculators can be sold per week at a price $P = 400 - 0.02n$ per calculator. Determine the value of n which maximises the profit.	A manufacturer makes calculators. The **total** cost to make these calculators is given by $T(x) = 100x + 200$ T(x) is the total cost in rands. x is the number of calculators made and sold. The selling price of **one** calculator is given by $S(x) = 400 - 0.02x$ S(x) is the selling price of one calculator in rands. Determine how many calculators must be sold for a maximum profit.

Original version (O): Assignment 5	Non-verbal version (N): Assignment 5
In a factory x units of a product A and y units of a product B are processed through two departments, D_1 and D_2. A requires 2 hours per unit in D_1 and 4 hours per unit in D_2. B requires 3 hours per unit in D_1 and 2 hours per unit in D_2. D_1 and D_2 have 60 and 80 hours respectively available each week. 5.1 Use the above information to write down the set of constraints in terms of x and y. 5.2 Represent these inequalities on the graph sheet provided and shade the feasible region. 5.3 If the profit margins are R30 and R40 for A and B respectively, write down the objective function. 5.4 Draw the optimal search line and hence write down the recommended product-mix that will maximise profit. 5.5 Determine the maximum profit. 5.6 What percentage of daily capacity will be utilized in each department?	Consider the following inequalities of a linear programming problem. $2x + 3y \leqslant 60$ $4x + 2y \leqslant 80$ $x \geqslant 0;\ y \geqslant 0$ 5.1 Draw these inequalities on a graph sheet. Shade the feasible region. 5.2 The objective function for this problem is given by $P = 30x + 40y$ 5.2.1 A maximum value of P is needed. Draw the graph of the objective function in an optimal position. 5.2.2 What are the values for x and y if P is a maximum? 5.3 Calculate the maximum value of P.